

Lorenzen’s free semilattices over integral domains

Stefan NEUWIRTH

In his habilitation (published in 1949), Paul Lorenzen proposed an analysis of the concept of ideal in terms of the preorder given by divisibility. He wrote: “An ideal system of a preordered set is nothing but an embedding into a semilattice”. In 1951, he showed that such an embedding is characterised by a single-statement entailment relation; an embedding into a distributive lattice is characterised by an entailment relation. In 1953, he used this to show how to embed a noncommutative preordered group into a lattice group.

In our work, done in collaboration with Thierry Coquand and Henri Lombardi, we describe Lorenzen’s lattice theoretic approach to multiplicative ideal theory in its last form (1952–1953). We also provide a complete and constructive description of embeddings of commutative partially ordered groups into distributive lattice groups in terms of “unbounded” entailment relations. In particular, we obtain a constructive proof of the so-called Lorenzen-Dieudonné theorem.

stefan.neuwirth@univ-fcomte.fr

Laboratoire de mathématiques de Besançon
Université de Franche-Comté
16 route de Gray
25030 Besançon CEDEX
France