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Heinrich Scholz and Paul Lorenzen

The connection between these two personalities is essentially documented by their correspondence, as well as by their correspondence with others. I have not been able to investigate how they first met but notice that Lorenzen's Ph.D. advisor is Helmut Hasse, a close friend of Scholz since the twenties.* In his letter to Carl Friedrich Gethmann of 14.1.1988, Lorenzen also mentions "several visits to Scholz in Münster" (see Gethmann, Phänomenologie, Lebensphilosophie und konstruktive Wissenschaftstheorie, 1991), so that one needs also to take into account this direct interaction. Special attention should be paid to Scholz's account of Lorenzen's stance in foundations of mathematics in his "Vorrede" to Scholz-Hasenjaeger, Grundzüge der mathematischen Logik, 1961).

A first period of their correspondence, documented by letters dating from 25.9.1943 to 9.7.1944, all by Lorenzen but for a few lines by Scholz of 17.5.1944, deals extensively with the introduction of real numbers, with Lorenzen's habilitation and his proof of consistency of elementary number theory (see Coquand-Neuwirth, Lorenzen's proof of consistency for elementary number theory, 2020), and shows Lorenzen's interest for the philosophy of Plato.

The second period of correspondence, documented by letters dating from 8.6.1946 to 17.2.1953 deals with a much greater variety of thems: Lorenzen's habilitation and proof of consistency of ramified type theory, the ramified type logic of Hans Hermes, Plato's philosophy, axioms for natural numbers, Scholz's "Anti-Stück", Max Bense's "Konturen einer Geistesgeschichte der Mathematik", the foundation of the Deutsche Gesellschaft für mathematische Logik und Grundlagenforschung,

* Lorenzen writes on 10.11.1939 to Hasse that he is making "group-axiomatic investigations for Professor Scholz".

Lorenzen's participation into the International congress of mathematicians at Cambridge (USA, 1950), the Hermes-Scholz-Logikbericht (1952), semantics with a distinction between content and form with a proposal to give a talk in Münster on "Zur Begründung einer formalen Semantik", that must have taken place in May 1952.

Lorenzen is often mentioned in the correspondence between Scholz and Bernays: it is Scholz who sent Bernays Lorenzen's proof of consistency for ramified type theory as soon as 11.12.1945; both discuss Lorenzen's constructive and operative mathematics; also the succession of Scholz at Münster is discussed: two candidates were on the short-list: Hans Hermes and Paul Lorenzen; Hans Hermes got the position.

But the most remarkable piece of evidence is a letter of Krull to Scholz, dated 18.4.1953, that contains a singular appraisal of Lorenzen's way of replacing the fictions used in working with well-ordering theorem.

Let us now focus on a few themes.

① The introduction of the real numbers.

In 1943-1944, Lorenzen discusses Tarski's axiom for real numbers as proposed in his "Einführung in die mathematische Logik und in die Methodologie der Mathematik (1937). The content of his letters can be found typewritten in the booklet "Gruppentheoretische Charakterisierung der reellen Zahlen" of the Mathematics library in Münster. He provides a more perspicuous system of axioms that allows for a particularly simple proof of independence for the axioms.

Lorenzen also explains to Scholz how the real numbers can be obtained in a semilattice-theoretic way by introducing formal suprema for sequences

of rational numbers with an upper bound. This method is known today as the "MacNeille completion".

Scholz's interest into this is old: as Sundholm points out in his "Heinrich Scholz between Frege and Hilbert", Scholz write in "Der Gottesgedanke in der Mathematik" (1934) that "continuum mathematics is in fact also still today and today the more no to be considered as a work of man, whose foundations are seriously contestable". For Scholz, this means that we need to continue to reflect upon the introduction of real numbers.

② Plato's philosophy

On 7.6.1944, Lorenzen write that "on the occasion of his glaring flop [he] wishes to distance himself from mathematics for a short while in order to try to understand something of the platonic philosophy — stimulated by the Natorp book". This book is certainly "Platos Ideenlehre". Lorenzen asks further Scholz for a German-Greek edition of the Phaedo. On 4.5.1947, he draws the following conclusion in a letter to Scholz: "It seems to me that constructivism does not at all contradict the platonic conception. For, that laws are something being, that of them can be known only what they are for themselves, that only they are truly being, that we can know only by our acting according to laws, by our participation in the laws — why should I contradict this. This is rather the most precise description of my conception.

③ Women

At working on Lorenzen, the only woman interacting with his mathematics is Rózsa Péter. But almost every letter also considers women: Scholz's wife Erna, called "die kleine Herrin", Lorenzen's wife Käthe and daughter Jutta, Bernays'

mother. Their hospitality is underlined: they do the housekeeping. They also play a professional role as typists. When Bernays' mother dies, Scholz is truly concerned and expresses the hope that one of Bernays' sisters will help him out in his daily life.

④ The frontier between philosophy and mathematics.

Scholz is a theologist and a philosopher who gets to understand that mathematical work needs to be done in order to progress in philosophy. Lorenzen is a mathematician who first notices that formal methods are helpful in his algebraic investigations, that if he puts them into a semilattice- or lattice-theoretic form, they lead to the same constructions as in logic. Then he comes to understand that mathematical problems like the foundations of analysis are difficult because of a bold philosophical stance, and that a philosophical clarification can lead to easier mathematics.

Scholz cross the frontier and enters mathematics; Lorenzen cross the frontier and enters philosophy. Both encounter problems of legitimacy: when Lorenzen proposes Scholz as referee for his habilitation, Krull answers that "Scholz, as a philosopher with mathematical interests, can only come into consideration as a coreferee". Lorenzen write the following to Krull: "In the foreground stands always the knowledge of the pure concepts. I naturally distance myself from pretending that this conception of mathematics is the right one, but I allow myself to claim that it is a rightful conception."

⑤ Control language and ortholanguage

Scholz and Lorenzen share the wish for a language that would allow for control. In his letter to Bernays of 8.2.1948, Lorenzen writes: "The goal of my investigations is the state that Hilbert has foreseen: 'that one must at all discussions about mathematics put the finger on the place where something should be wrong'" (cited imprecisely). It is the same goal that Scholz calls 'controllable language'.