# The philosophy of dynamical algebra

The title is an allusion to the book Philosophie de l'algèbre by Jules Vuillemin of the 60s. It deals e.g. with the meaning of solving a polynomial equation. Dynamical algebra is also about this, but let us speak about this later. There are three words in the title; I don't feel the need to speak about the last one, «algebra», because it is not really fundamental here, it is just the domain I shall speak about; but there is something I wish to stress: as algebra is about solving equations, it is natural to imagine it as a concrete business that deals with algorithms — so it is surprising that algebra has become so abstract (abstract structures, abstract level of discourse) and so noneffective (proving existence without effective access to the solution because one in fact proves the absurdity of non-existence; using noneffective procedures, i.e. procedures that we cannot effectively apply, prominently an infinity of choices, but also to distinguish indiscernibles). The core of this course will deal with the epithet «dynamical», so please wait. I presently feel the urge of justifying the word «philosophy». In my opinion, philosophy need not be a discourse about algebra, I rather call for a philosophy of algebra as a specific component of algebra. But which component is it? In order to state my point, let me use the words of this school: «proof» and «computation». In French, «comput» is used only for computing the date of Easter, and for this you combine certain data of the ephemerids: the date of the equinox, the date of full moon, and you get a certain result. So to compute is just to follow certain rules that transform certain data into other data. There are also interesting reflections to be had here, but let us presume that this is perfectly clear to us, and consider computation as formal. But how do we know what the computation is about? One way of knowing is to spell out what the result of the computation is supposed to provide and to prove that the computation is actually providing it. (One speaks about showing the correction of an algorithm.) In doing so, one is in a specific mindset of transporting the truth of the initial data through every step seen as a kind of dangerous jump in which evidence must be preserved by convincing ourselves that what ought to be preserved is actually preserved. How does philosophy step in here? I don't believe that there can be an exhaustive answer to this question, so I will just try to collect a few occurrences: ① what is the computation actually about? Is it possible to clarify how the object of the actual discourse is replaced by another one, by a different kind of dangerous jump in which the transport ought to be provided in both directions? This jump is most prominently the «abstraction» in which a certain structure is considered to allow certain objects to be freed from certain contingencies and the relationships between them to be revealed. In a nutshell, philosophy can be about the ontology of mathematical objects. ② What is the computation actually doing? What meaning do the images that we rely on for this understanding actually have? There are spatial images, temporal processes, graph-like structures, most prominently diagrams, combinatorics of signs. How do they acquire their meaning? There is practice, which is in some sense an adequation of the mental structure of the mathematician with the structure under study. ③ Philosophy can also be about the nature of this relationship and especially about keeping alive, i.e. in process, in change, in becoming, thus about the position and nature of the mathematician.

All that has been addressed has actually also become part of mathematics itself in a process that has been called reflection: our own way of understanding is meaningful to us and leads to elaborating specific mathematics that express this meaning. Thus, to do philosophy of algebra, of mathematics, is about being a mathematician in the search of understanding his own activity, and this is mathematically fruitful; it is at least a way of approaching the progress of mathematics, and also a way of empowerment of the mathematician.